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XXVI. *Researches in Physical Astronomy.* By J. W. LUBBOCK, Esq. V.P.
and Treas. R.S.

Read June 21, 1832.

On the development of R .

IN the following method of developing the disturbing function, the coefficients of the inequalities corresponding to any given order are expressed in terms of the coefficients of the inferior orders; so that, for example, the coefficients of the terms in the disturbing function multiplied by the squares of the eccentricities, are given analytically by means of the coefficients of those independent of the eccentricities, and of those multiplied by their first powers. As the theorems to which this method gives rise, are of great simplicity, I trust they will not be thought unworthy attention. By their means and with the assistance of the table given in my Lunar Theory, the expressions may be obtained, which are necessary for the development of R , as far as the fourth powers of the eccentricities inclusive; it may easily be carried to any extent, and the expressions given by BURCKHARDT in the *Mémoires de l'Institut*, 1808, may be verified without difficulty. This method is peculiarly advantageous in the lunar theory, and for the terms in R dependent on powers of the eccentricities above the squares; for the expression thus obtained for the coefficients of the terms dependent on the squares and products of the eccentricities in the planetary theory, is by no means so simple or so convenient for numerical calculation as that given in the *Phil. Trans.* 1831, p. 295. A similar method is applicable to the terms dependent on the inclinations.

Let $R = R_0 + e^2 R_0' + e_1^2 R_0'' + \&c.$

$$+ \{ R_1 + e^2 R_1' + e_1^2 R_1'' + \&c. \} \cos (i n t - i n_1 t)$$

[1]

$$+ \{ R_2 + e^2 R_2' + e_1^2 R_2'' + \&c. \} e \cos (n t - \varpi)$$

[2]

$$+ \{R_3 + e^2 R_3 + e_1^2 R_3 + \&c\} e \cos (i n t - i n_1 t - n t + \varpi)$$

$$+ \&c. \quad [3]$$

where the indices are as follows, and the same as in my Lunar Theory, merely writing the indeterminate i instead of the number 2.

0	0	21	$it - 3x$	42	$it - 3x - z$
1	it	22	$it + 3x$	43	$it + 3x + z$
2	x	23	$2x + z$	44	$3x - z$
3	$it - x$	24	$it - 2x - z$	45	$it - 3x + z$
4	$it + x$	25	$it + 2x + z$	46	$it + 3x - z$
5	z	26	$2x - z$	47	$2x + 2z$
6	$it - z$	27	$it - 2x + z$	48	$it - 2x - 2z$
7	$it + z$	28	$it + 2x - z$	49	$it + 2x + 2z$
8	$2x$	29	$x + 2z$	50	$2x - 2z$
9	$it - 2x$	30	$it - x - 2z$	51	$it - 2x + 2z$
10	$it + 2x$	31	$it + x + 2z$	52	$it + 2x - 2z$
11	$x + z$	32	$x - 2z$	53	$x + 3z$
12	$it - x - z$	33	$it - x + 2z$	54	$it - x - 3z$
13	$it + x + z$	34	$it + x - 2z$	55	$it + x + 3z$
14	$x - z$	35	$3z$	56	$x - 3z$
15	$it - x - z$	36	$it - 3z$	57	$it - x + 3z$
16	$it + x - z$	37	$it + 3z$	58	$it + x - 3z$
17	$2z$	38	$4x$	59	$4z$
18	$it - 2z$	39	$it - 4x$	60	$it - 4z$
19	$it + 2z$	40	$it + 4x$	61	$it + 4z$
20	$3x$	41	$3x + z$		

$$r = 1 + \frac{e^2}{2} - e \left(1 - \frac{3e^2}{8}\right) \cos x - \frac{e^2}{2} \left(1 - \frac{2e^2}{3}\right) \cos 2x + \frac{9}{8} e^3 \cos 3x + \frac{4}{3} e^4 \cos 4x$$

$$\frac{dr}{de} = e - \left(1 - \frac{9}{8} e^2\right) \cos x - e \left(1 - \frac{4e^2}{3}\right) \cos 2x + \frac{27}{8} e^3 \cos 3x + \frac{16}{3} e^3 \cos 4x$$

$$\frac{dr}{r de} = \frac{e}{2} \left(1 + \frac{e^2}{4}\right) - \left(1 - \frac{9}{8} e^2\right) \cos x - \frac{3}{2} e \left(1 - \frac{11}{9} e^2\right) \cos 2x$$

$$- \frac{17}{8} e^2 \cos 3x - \frac{71}{24} e^3 \cos 4x$$

$$\frac{d\lambda}{de} = 2 \left(1 - \frac{3e^2}{8}\right) \sin x + \frac{5}{2} e \left(1 - \frac{28}{15} e^2\right) \sin 2x + \frac{13}{4} e^2 \sin 3x + \frac{103}{24} e^2 \sin 4x$$

$$\frac{dR}{de} = \frac{dR}{dr} \frac{dr}{de} + \frac{dR}{d\lambda} \frac{d\lambda}{de}$$

$$= \frac{r}{dr} \frac{dR}{r de} + \frac{dR}{d\lambda} \frac{d\lambda}{de}$$

$$\frac{r \, d R}{d r} = \frac{a \, d R}{d a} \qquad \frac{d R}{d \lambda} = -i R^*$$

Multiplying by means of Table II. Phil. Trans. 1831, p. 238, we find

$$R_2 = -\frac{a \, d R_0}{d a} \qquad R_3 = -\frac{a \, d R_1}{2 \, d a} - i R_1 \qquad R_4 = -\frac{a \, d R_1}{2 \, d a} + i R_1$$

$$2 R_8 = -\frac{a \, d R_2}{2 \, d a} - \frac{3 a \, d R_0}{2 \, d a}$$

$$2 R_9 = -\frac{a \, d R_3}{2 \, d a} - i R_3 - \frac{3 a \, d R_1}{4 \, d a} - \frac{5 i R_1}{4}$$

$$2 R_{10} = -\frac{a \, d R_4}{2 \, d a} + i R_4 - \frac{3 a \, d R_1}{4 \, d a} + \frac{5 i R_1}{4}$$

These equations may be formed at once from the Table by inspection, taking care to write R with the sign $+$ in the term multiplied by i when the index is found in the upper line in the Table, as in the case of the argument (10); and with the sign $-$ when in the lower, as in the case of the argument (9). The term multiplied by $\frac{a \, d R}{d a}$ always takes its sign from the factor arising from $\frac{d r}{r \, d e}$. In what precedes, i is any positive whole number.

By means of the Tables, any term in R depending on the eccentricities may be found at pleasure, and the development given in the Phil. Trans. 1831, p. 263, may be verified with great facility; thus

$$4 R_{38} = -\frac{a \, d R_{20}}{2 \, d a} - \frac{3 a \, d R_8}{4 \, d a} - \frac{17 a \, d R_2}{16 \, d a} - \frac{71 a \, d R_0}{24 \, d a}$$

I find on reference to the development in question

$$R_{38} = \frac{a^2}{24 a_i^3} \qquad R_{20} = \frac{a^2}{16 a_i^3} \qquad R_8 = \frac{a^2}{8 a_i^3} \qquad R_2 = \frac{a^2}{2 a_i^3} \qquad R_0 = -\frac{a^2}{4 a_i^3}$$

whence

$$a \frac{d R_{20}}{d a} = \frac{a^2}{8 a_i^3} \qquad a \frac{d R_8}{d a} = \frac{a^2}{4 a_i^3} \qquad a \frac{d R_2}{d a} = \frac{a^2}{a_i^3} \qquad a \frac{d R_0}{d a} = -\frac{a^2}{2 a_i^3}$$

which values satisfy the equation above, for

$$\frac{4}{24} = \frac{1}{2 \cdot 8} - \frac{3}{4 \cdot 4} - \frac{17}{16} + \frac{71}{24 \cdot 2}$$

By successive substitutions in the expressions which have been given, it is

* This is only a method of notation as regards the coefficients, which will be easily understood.

obvious that they may be reduced so as to contain only the quantity R_1 and the differential coefficients of this quantity with respect to a and a_1 .

Thus

$$\begin{aligned}
 R_4 &= -\frac{a \, d \, R_1}{2 \, d \, a} + i R_1 \\
 2 \, R_{10} &= -\frac{a \, d \, R_4}{2 \, d \, a} + i R_4 - \frac{3 \, a \, d \, R_1}{4 \, d \, a} + \frac{5 \, i \, R_1}{4} \\
 &= -\frac{1}{2} \left\{ -\frac{a^2 \, d^2 \, R_1}{2 \, d \, a^2} - \frac{a \, d \, R_1}{2 \, d \, a} + i a \frac{d \, R_1}{d \, a} \right\} \\
 &\quad - \frac{i a \, d \, R_1}{2 \, d \, a} + i^2 R_1 - \frac{3 \, a \, d \, R_1}{4 \, d \, a} + \frac{5 \, i \, R_1}{4} \\
 R_{10} &= \frac{a^2 \, d^2 \, R_1}{8 \, d \, a^2} - \frac{(2i+1) \, a \, d \, R_1}{4 \, d \, a} + \frac{(4i^2+5i) \, R_1}{8}
 \end{aligned}$$

Changing the sign of i , we get

$$R_9 = \frac{a^2 \, d^2 \, R_1}{8 \, d \, a^2} + \frac{(2i-1) \, a \, d \, R_1}{4 \, d \, a} + \frac{(4i^2-5i) \, R_1}{8}$$

which accords with the expression (for $N^{(0)}$) given in the *Théor. Anal.* vol. i. p. 463.

$$\begin{aligned}
 3 \, R_{22} &= -\frac{a \, d \, R_{10}}{d \, a} + i R_{10} - \frac{3}{4} \frac{a \, d \, R_4}{d \, a} + \frac{5i}{4} R_4 - \frac{17}{16} \frac{a \, d \, R_1}{d \, a} + \frac{13i}{8} R_1 \\
 &= -\frac{1}{2} \left\{ \frac{a^2 \, d^3 \, R_1}{8 \, d \, a^3} + \frac{a^2 \, d^2 \, R_1}{4 \, d \, a^2} - \frac{(2i+1) \, a^2 \, d^2 \, R_1}{4 \, d \, a^2} - \frac{(2i+1) \, a \, d \, R_1}{4 \, d \, a} + \frac{(4i+5) \, i \, d \, R_1}{8 \, d \, a} \right\} \\
 &\quad + i \left\{ \frac{a^2 \, d \, R_1}{8 \, d \, a^2} - \frac{(2i+1) \, a \, d \, R_1}{4 \, d \, a} + \frac{(4i+5) \, i \, R_1}{8} \right\} \\
 &\quad - \frac{3}{4} \left\{ -\frac{a^2 \, d^2 \, R_1}{2 \, d \, a^2} - \frac{a \, d \, R_1}{2 \, d \, a} + \frac{i a \, d \, R_1}{d \, a} \right\} \\
 &\quad + \frac{5i}{4} \left\{ -\frac{a \, d \, R_1}{2 \, d \, a} + i R_1 \right\} - \frac{17}{16} \frac{a \, d \, R_1}{d \, a} + \frac{13}{8} R_1 \\
 R_{22} &= \frac{1}{48} \left\{ (26i+30i^2+8i^3) \, R_1 - (9+27i+12i^2) \frac{a \, d \, R_1}{d \, a} \right. \\
 &\quad \left. + (6i+6) \frac{a^2 \, d^2 \, R_1}{d \, a^2} - \frac{a^3 \, d^3 \, R_1}{d \, a^3} \right\}
 \end{aligned}$$

Changing the sign of i , we get

$$R_{21} = -\frac{1}{48} \left\{ (26i - 30i^2 + 8i^3) R_1 + (9 - 27i + 12i^2) a \frac{dR}{da} \right. \\ \left. + (6i - 6) \frac{a^2 d^2 R_1}{d a^2} + \frac{a^3 d^3 R_1}{d a^3} \right\}$$

which agrees with the expression given by BURCKHARDT for $(M^{(0)})$, *Memoires de l'Institut*, 1808, *Second Semestre*, p. 39.

Similarly

$$2R_{51} = \frac{a^2 d^2 R_{19}}{4 d a^2} + \frac{(2i - 1) a d R_{19}}{2 d a} + \frac{(4i^2 - 5i) R_{19}}{4} \\ 2R_{19} = \frac{a_i^2 d^2 R_1}{4 d a_i^2} + \frac{(2i - 1) a_i d R_1}{2 d a_i} + \frac{(4i^2 - 5i) R_1}{4}$$

If $i = 2$,

$$2R_{51} = \frac{a^2 d^2 R_{19}}{4 d a^2} + \frac{3}{2} \frac{a d R_{19}}{d a} + \frac{3}{2} R_{19} \\ 2R_{19} = \frac{a_i^2 d^2 R_1}{4 d a_i^2} + \frac{3}{2} \frac{a_i d R_1}{d a_i} + \frac{3}{2} R_1 \\ R_1 = -\frac{b_{1,2}}{a_i} = -\frac{3a^2}{4a_i^3} - \frac{3 \cdot 5}{2 \cdot 4 \cdot 6} \frac{a^4}{a_i^5} - \frac{3 \cdot 3 \cdot 5 \cdot 7}{2 \cdot 4 \cdot 4 \cdot 6 \cdot 8} \frac{a^6}{a_i^7} - \&c.$$

In the Lunar Theory, the higher terms may be neglected; and taking $R_1 = -\frac{3a^2}{4a_i^3}$, it is evident that R_{19} and R_{51} are each equal to zero. This theorem, however, cannot be extended to the other terms, and therefore in the Planetary Theory the coefficient corresponding to the argument $2t - 2x + 2z$ or $2\varpi - 2\varpi_p$, in the development of R , (which term is important as regards the secular inequalities,) does not vanish.

If the coefficients of the n th argument in the expressions for $\frac{a}{r}$ and λ be called r_n and λ_n , the Table which has been used for the preceding multiplications may also be used (when the square of the disturbing force is neglected,) for the integration of the equations

$$\frac{d^2 r^2}{2 dt^2} - \frac{\mu}{r} + \frac{\mu}{a} + 2 \int dR + r \frac{dR}{dr} = 0$$

and

$$\frac{d\lambda}{dt} = \frac{h}{r^2} - \frac{1}{r^2} \int \frac{dR}{d\lambda} dt \\ - \frac{d^2 r^3}{dt^2} \delta \frac{1}{r} - \mu \delta \frac{1}{r} + 2 \int dR + \frac{r dR}{dr} =$$

	62	65	66	77	78	146	149	150	161	162			62	65	66	77	78	146	149	150	161	162	
62 { ...	2	-2	146	150	62	79 { 9	...	3	...	1	163	151	147	79
63 { 1	3	...	4	147	151	63	80 { 9	...	3	...	1	164	152	148	80
64 { 1	...	3	148	152	64	81 { 10	...	4	...	1	165	153	147	81
65 { 2	149	-146	65	82 { 10	...	4	...	1	166	154	148	82
66 { 2	150	146	66	83 { 11	...	5	167	155	83
67 { 3	...	1	151	147	67	84 { 11	...	5	168	156	84
68 { 3	1	152	148	68	85 { 12	...	6	169	157	85
69 { 4	153	147	69	86 { 12	...	6	170	158	86
70 { 4	...	1	154	148	70	87 { 13	...	7	171	159	87
71 { 5	155	71	88 { 13	...	7	172	160	88
72 { 5	156	72	89 { 14	-5	173	-156	89
73 { 6	157	73	90 { 14	...	-5	174	-155	90
74 { 6	158	74	91 { 15	...	7	175	159	91
75 { 7	159	75	92 { 15	...	7	176	160	92
76 { 7	160	76	93 { 16	...	6	177	157	93
77 { 8	2	161	149	-146	77	94 { 16	...	6	178	158	94
78 { 8	...	2	162	150	146	78												